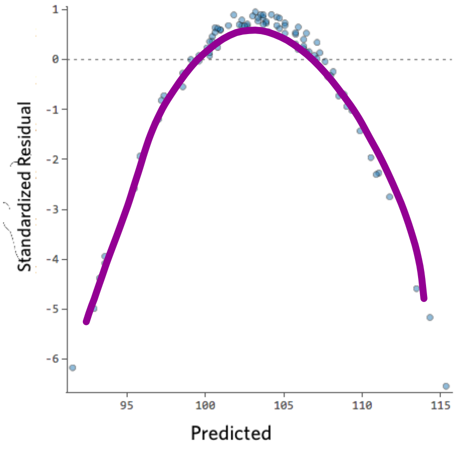
Warm-Up (10/23/17)

From the residual plot below, does this represent a linear or a nonlinear scatter plot? Explain your reasoning.

Because this makes a parabola, then it's scatter plot is Non linear.



Module 3: Lesson 9 & Lesson 10

Representing, Naming, and Evaluating Functions

Lesson Summary:

- In this lesson we define a function as a correlation between two sets where every input has exactly one output.

-Domain and range are defined informally, but there are formal definitions that are offered by the textbook (Module 3, pg. 53)

- Students will be able to check for a function through means of sets, ordered pairs, and graphs.

- Students will be able to evaluate a function through sets, function notation, and graphs.

What is a function?

A function is a correspondence between two sets, *X* and *Y*, in which each element of *X* is matched to one and only one element of *Y*. The set *X* is called the domain of the function.

In other words, a function is a RULE in which every X has EXACTLY ONE Y. If one X has 2 Y's then you do not have a function. (Note that one Y can have 2 X's).

Definitions

X

Input: a value that goes into the function

Y

Output: a value calculated from the input of a function

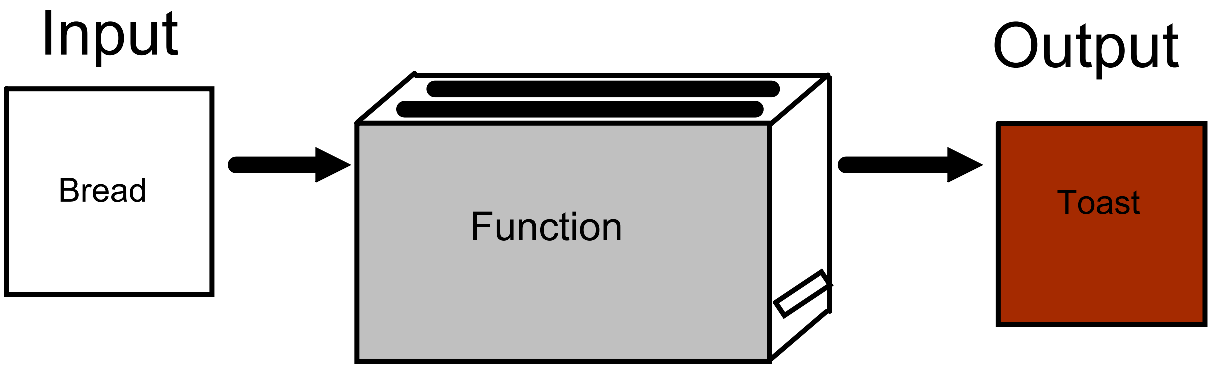
All X values

Domain: all possible inputs of a function

All y-values

Range: all possible outputs of a function

One way to think about functions is to think about a toaster:



1. The function (or the "rule") is the toaster.

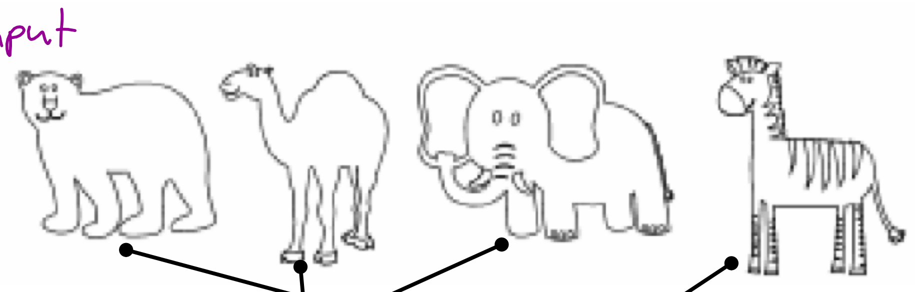
2. Our input is the bread.

3. One slice of bread goes into the toaster, and we get one piece of toast as a result (the output). In other words, our input has exactly one output.

Before we introduce numbers into our functions, let's think about a few of these examples to determine if they could be representative of a function.

Match each picture to the correct word by drawing an arrow from the word to the picture.

Input



Output

Elephant Camel Zebra Polar Bear

In this example above we matched the pictures of the animals to exactly one animal name. So this is representative of a function.

2. The assignment of a team of football players to jersey numbers.

Yes, because each football player gets exactly 1 jersey #.

3. The assignment of states and capitals.

Yes, b/c each state has exactly one capital.

4. The assignment of states and cities.

No, b/c a state can have multiple cities.

5. The assignment of zip codes to residents.

No, because a zip code contains more than 1 resident.

The Anatomy of a Function

You may see functions expressed the following ways:

Indicates a function

Domain

$$f : X \to Y$$

Range

This is read as "the function, f, is mapped from the sets X to Y".

NOT Multiplication

Indicates a function

Input

output

$$f(x) = x + 1$$

This is read as "f of x equals x plus one"

This is the most common way you will see a function written.

Other common variables used are *g* and *h* when we are dealing with multiple functions in one problem.

Please note:

AN EQUATION IS **NOT** A FUNCTION!!!!

An equation can be used to represent a function, or a function may be written like an equation and may be treated like an equation. However, by definition, equations are NOT functions.

How to check for a function: Example 1

Determine whether or not the following set of ordered pairs represents a function.

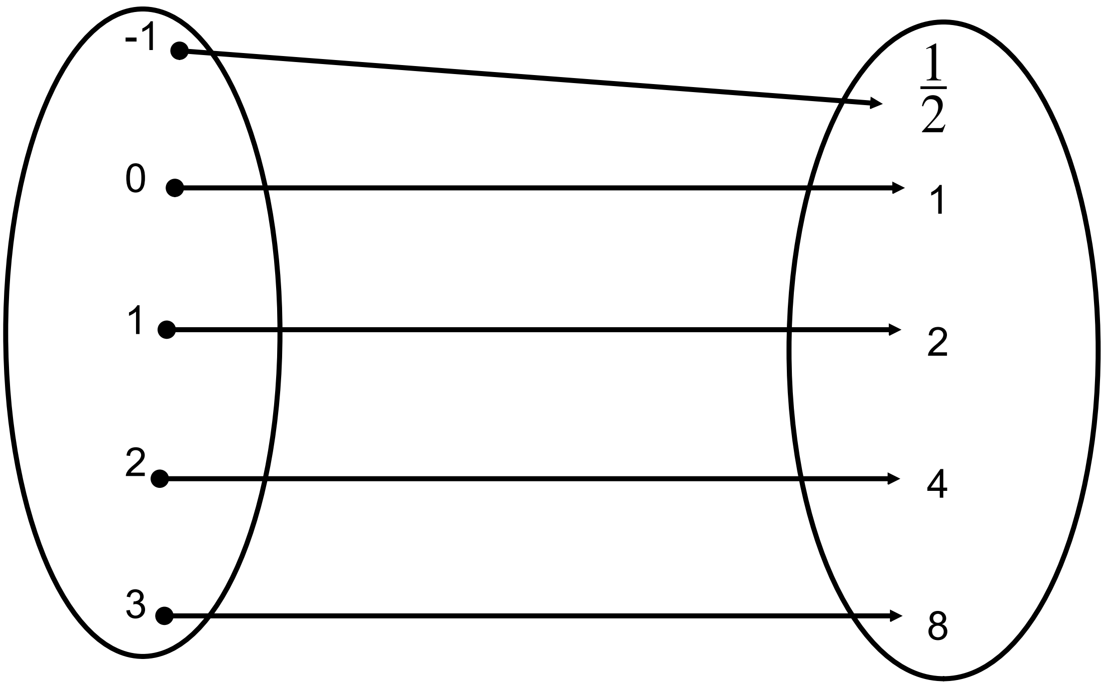
(x, y)

$$\left\{(0, 1), (1, 2), \left(-1, \frac{1}{2}\right), (2, 4), (3, 8)\right\}$$

To determine this we will make two columns to organize our information. This way we can see if every X has exactly one Y.

Below we show the correspondence between each ordered pair drawing arrows to "map" them to each other. Every x is "mapped" to its corresponding y from our data set.

Domain (x-values) Range (y-values)



As observed above, each input (x) has a correspondence with EXACTLY ONE output (y). Because of this, the above ordered pairs would be representative of a function.

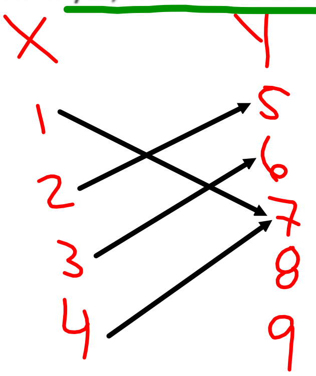
How to check for a function: Example 2

Let $X = \{1, 2, 3, 4\}$ and $Y = \{5, 6, 7, 8, 9\}$. f and g are defined below.

$f: X \to Y$ $\qquad$ $g: X \to Y$

$f = \{(1,7), (2,5), (3,6), (4,7)\}$ $\qquad$ $g = \{(1,5), (2,6), (1,8), (2,9), (3,7)\}$

Is f a function? If yes, what is the domain, and what is the range? If no, explain why f is not a function.

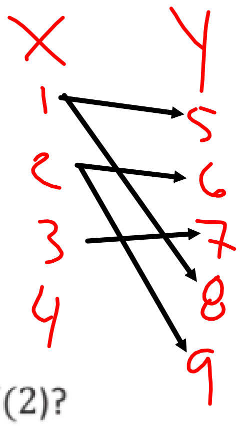


Yes, f is a function because each input (x) has exactly one output (y).

X ← Domain: $\{1, 2, 3, 4\}$

Y ← Range: $\{5, 6, 7\}$

Is g a function? If yes, what is the domain and range? If no, explain why g is not a function.



Not a function because two of our inputs(x) have more than one output(y)

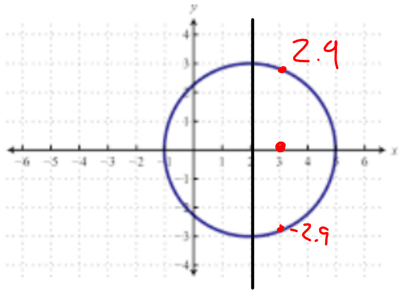
What is $f(2)$?

$x=2, y=?$

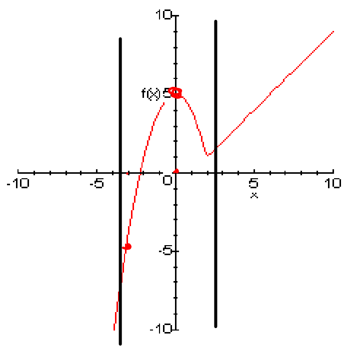
$\boxed{f(2)=5}$

How to check for a function: Example 3

Determine whether or not the following two graphs are functions.



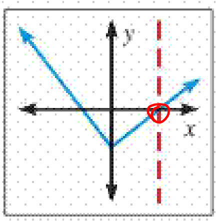
This is NOT a function because there are many inputs that have two outputs



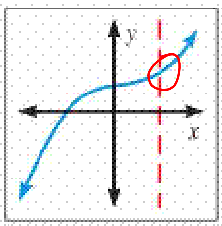
This is a function because every input has exactly one output.

For simple graphs of functions we can check to see if they are functions by what is known as the "vertical line test". For this test we just drop a vertical line on the graph and if the graph crosses our vertical line twice, then the graph is not representative of a function.

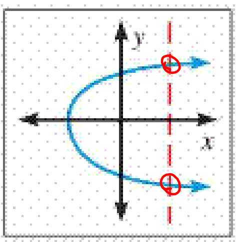
Here are some examples:



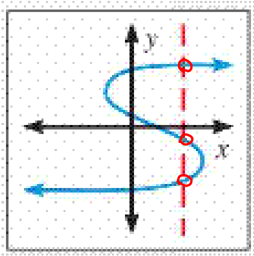
function



function



not a function



not a function

Evaluating Functions

To evaluate a function we take a given input, and substitute it into our function to obtain an output.

Example:

Consider the function $f(x) = 2x - 1$.

Evaluate $f(6)$

$f(6) = 2(6) - 1$

$= 12 - 1$

$\boxed{f(6) = 11}$

If $f(x) = 5$, what is the value for x?

$f(x) = 2x - 1$

$5 = 2x - 1$

$+1 \qquad +1$

$6 = 2x$

$\boxed{x = 3}$

Example:

The following is a graph of a function, f(x).

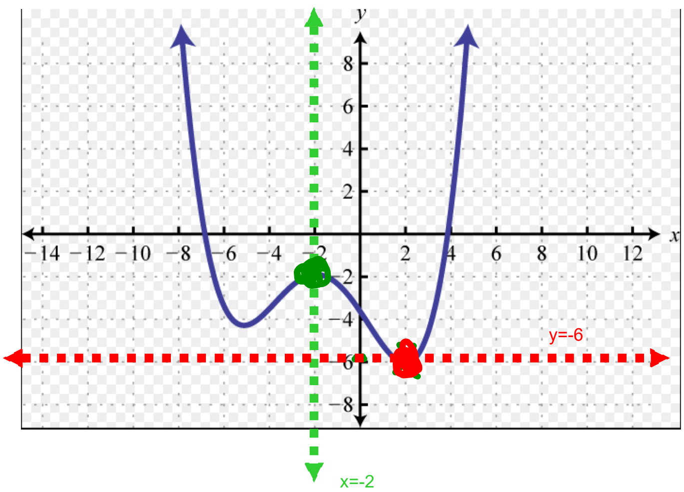
a. Evaluate the function for $f(-2)$

$x = -2$, $y = ?$

$f(-2) = -2$

b. If $f(x) = -6$, what is the value for the input?

$x = 2$



For (a) we are given in input (x) and wish to find its corresponding output (y). This is done so by finding x=2 on the graph and locating where the graph has a y-value for x=2. (This is done in green)

For (b) we are given the output (y) and are tasked to find its input (x). This is done so by locating the value for y=-6 on the y-axis and checking where the graph crosses y=-6 (this is done in red). Note that it is possible to have more than one input for this (you may have more than one x-value for a question like this).

Final Notes

The following notation will be used to define functions going forward. If a domain is not specified, it is assumed to be the set of all real numbers.

For the squaring function, we say Let $f(x) = x^2$.

For the exponential function with base 2, we say Let $f(x) = 2^x$.

When the domain is limited by the expression or the situation to be a subset of the real numbers, it must be specified when the function is defined.

For the square root function, we say Let $f(x) = \sqrt{x}$ for $x \geq 0$.

Depending on the context, one either views the statement "$f(x) = \sqrt{x}$" as part of defining the function f or as an equation that is true for all x in the domain of f or as a formula.