Warm Up (11/15/17)

Generate the formula for the following sequence of numbers:

1st 2nd 3rd . . . .

5, 8, 11, 14, 17, ...

$a_1 = 5$

$d = 3$

$a_n = a_1 + d(n-1)$

$a_n = 5 + 3(n-1)$

Module 3: Lesson 2

Recursive Formulas for Sequences

Lesson Summary

RECURSIVE SEQUENCE: An example of a *recursive sequence* is a sequence that (1) is defined by specifying the values of one or more initial terms and (2) has the property that the remaining terms satisfy a recursive formula that describes the value of a term based upon an expression in numbers, previous terms, or the index of the term.

An explicit formula specifies the n^{th} term of a sequence as an expression in n.

A recursive formula specifies the n^{th} term of a sequence as an expression in the previous term (or previous couple of terms).

What is a recursive formula?

With an explicit formula we can find the value of any term in a sequence without needing to know the value of the term before it.

A recursive formula is a formula that relies on knowing the previous term of the sequence.

General Form of a Recursive Formula

$$a_n = a_{n-1} + d$$

"Current term"

Previous Term

"The difference"

- Given at least one term

Ex $a_1 = 10$; $a_6 = 4$

- Given the interval for n

Ex $n \geq 2$

Example 1

Consider the sequence given by the formula $a_n = a_{n-1} - 5$ where $a_1 = 12$ and $n \geq 2$.

a. Write out the first 5 terms of the sequence

$a_1 = 12$

$a_2 = a_{2-1} - 5 = a_1 - 5 = 12 - 5 = 7$

$a_3 = a_{3-1} - 5 = a_2 - 5 = 7 - 5 = 2$

$a_4 = a_{4-1} - 5 = a_3 - 5 = 2 - 5 = -3$

$a_5 = -8$

b. Write an explicit formula from the 5 terms found in part (a).

$12, 7, 2, -3, -8, \ldots$

This sequence is Arithmetic. So we use the formula $a_n = a_1 + d(n-1)$

What is the first term? $a_1 = 12$

What is the difference ("the pattern")?

$d = -5$

$a_n = 12 - 5(n-1)$

c. Find a_6 and a_{100} of the sequence.

$a_6 = 12 - 5(6-1)$
$= 12 - 5(5)$
$= 12 - 25$
$a_6 = -13$

$a_{100} = 12 - 5(100-1)$
$= 12 - 5(99)$
$= 12 - 495$
$a_{100} = -483$

Example 2

Consider the sequence given by the formula $a_n = a_{n-1} - 8$ where $a_1 = 9$ and $n \geq 2$.

previous term

a. Write out the first 5 terms of the sequence

$a_1 = 9$ (Given)

$a_2 = a_{2-1} - 8 = a_1 - 8 = 9 - 8 = 1$

$a_3 = a_{3-1} - 8 = a_2 - 8 = 1 - 8 = -7$

$a_4 = a_{4-1} - 8 = a_3 - 8 = -7 - 8 = -15$

$a_5 = a_{5-1} - 8 = a_4 - 8 = -15 - 8 = -23$

b. Write an explicit formula from the 5 terms found in part (a).

$9, 1, -7, -15, -23, \ldots$

Since this is Arithmetic, we use the formula

$a_n = a_1 + d(n-1)$

$a_1 = 9$ (1st term of the sequence)

$d = -8$ (the pattern of the sequence)

$$a_n = 9 - 8(n-1)$$

Example 3

Consider the sequence given by the formula $a_n = a_{n-1} + 4$ where $a_1 = 7$ and $n \geq 2$.

a. Write out the first 5 terms of the sequence

$a_1 = 7$

$a_2 = a_{2-1} + 4 = a_1 + 4 = 7 + 4 = 11$

$a_3 = a_{3-1} + 4 = a_2 + 4 = 11 + 4 = 15$

$a_4 = a_{4-1} + 4 = a_3 + 4 = 15 + 4 = 19$

$a_5 = a_{5-1} + 4 = a_4 + 4 = 19 + 4 = 23$

b. Write an explicit formula from the 5 terms found in part (a).

7, 11, 15, 19, 23, ...

$a_1 = 7$

$d = 4$

Since this is an Arithmetic Sequence we use $a_n = a_1 + d(n-1)$

$a_n = 7 + 4(n-1)$