Module 3: Lesson 1 & Lesson 3

Introduction to Sequences

Lesson Summary

Think of a sequence as an ordered list of elements. Give an explicit formula to define the pattern of the sequence. Unless specified otherwise, find the first term by substituting 1 into the formula.

Two types of sequences were studied:

ARITHMETIC SEQUENCE: A sequence is called *arithmetic* if there is a real number d such that each term in the sequence is the sum of the previous term and d.

GEOMETRIC SEQUENCE: A sequence is called *geometric* if there is a real number r such that each term in the sequence is a product of the previous term and r.

Opening Exercise:

What is the next term of the following list of numbers?

$$2 \xrightarrow{+2} 4 \xrightarrow{+2} 6 \xrightarrow{+2} 8, \ldots \quad 10$$

What pattern does the list of numbers seem to follow?

We add two to the previous term

What is a sequence?

A sequence is an ordered list of elements, such as numbers, that follows a particular pattern. Each entry in a sequence is usually referred to as an "element" or a "term" of the sequence.

A sequence is represented by a function where n is our variable and where our domain is the positive integers.

Notation

While sequences can be represented through function notation, we more commonly represent it through *subscript notation*.

- function
- variable, input, is n

$$f(n) = a_n$$

→ Sequence

→ Index

Note: The Index tells us where we are in the sequence. For example, if n=1 we are looking for the value of the first term. If n=10 we are looking for the value of the 10th term.

This is read as "a sub n" where n acts like our input for a function.

We use subscript notation to indicate that we are working with a sequence, not just a general function.

Example:

Find the next three terms of the following sequence:

$4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13$, 16, ... 19, 22, 25

1 2 3 4 5 6 7 8

20th term? 61

Finding patterns to sequences

When we define sequences we find the "pattern" just like finding a "rule" to a function.

Consider our previous example:

+3 +3 +3 +3

4, 7, 10, 13, 16, ... 19, 22, 25, ...

What was our pattern that we observed? That is, what did we do to the previous term of our sequence in order to get to the next term?

We added 3 to every previous term

Different kinds of sequences

Arithmetic sequence: A sequence in which the terms increase or decrease by a fixed amount.

Ex: $1 \xrightarrow{+2} 3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9$, ...

In this example, we are adding 2 to the previous term.

Geometric sequence: A sequence in which the terms increase or decrease by a fixed non-zero MULTIPLE (this is called the "common ratio").

Ex: $1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81$, ... $\xrightarrow{\times 3}$ 243

In this example we are multiplying the previous term by 3 in order to get the value of the next term.

Example:

Find the following terms of the given arithmetic sequence:

1 2 3 4 5 6 7 8

3, 5, 7, 9, 11, 13, 15, 17 ...

+2 +2 +2 +2 +2 +2 +2

$a_9 = 19$ +2

$a_{10} = 21$ +2

$a_{11} = 23$ +2

$a_{12} = 25$

$a_1 = 3$ $a_4 = 9$

1st term

$a_8 = 17$ $a_{12} = 25$

Example:

Find the following terms of the given geometric sequence:

1 2 3 4 5 6

4, 8, 16, 32, 64, 128, 256, ...

×2 ×2 ×2

$a_1 = 4$

1st Term

$a_2 = 8$

2nd term

$a_6 = 128$

$a_8 = 512$

Example:

Find the 1st, 5th, and 12th terms of the following arithmetic sequence:

$\boxed{a_1 = 3}$

$\boxed{a_5 = 11}$

$\boxed{a_{12} = 25}$

$$a_n = 3 + 2(n - 1)$$

$$a_1 = 3 + 2(1 - 1)$$
$$= 3 + 2(0)$$
$$= 3 + 0$$
$$= 3$$

$$a_5 = 3 + 2(5 - 1)$$
$$= 3 + 2(4)$$
$$= 3 + 8$$
$$= 11$$

$$a_{12} = 3 + 2(12 - 1)$$
$$= 3 + 2(11)$$
$$= 3 + 22$$
$$= 25$$

Example:

Find the 1st, 3rd, and 4th terms of the following geometric sequence:

$$a_n = 3^{n-1}$$

$\boxed{a_1 = 1}$ $\boxed{a_3 = 9}$ $\boxed{a_4 = 27}$

$a_1 = 3^{1-1}$
$= 3^0$
$= 1$

$a_3 = 3^{3-1}$
$= 3^2$

$a_4 = 3^{4-1}$
$a_4 = 3^3$

Explicit formula for an **ARITHMETIC** sequence

(Use if adding or subtracting terms)

$$a_n = a_1 + d(n-1)$$

1st Term

"The difference" or The "pattern"

Let's take a look at a previous example

Generate the explicit formula for the following sequence:

+3 +3 +3 +3

4, 7, 10, 13, 16, ...

1st Term

What is our first term? $a_1 = 4$

What is the "pattern" of our sequence?

We are adding 3 to the previous terms.

So, d=3

Now we will use these values to create our explicit formula:

1. Write out the formula that we're using

2. Plug in our values for d and a_1

$$a_n = a_1 + d(n-1)$$

$$\boxed{a_n = 4 + 3(n-1)}$$

Explicit formula for a **GEOMETRIC** sequence

(Use if multiplying or dividing terms)

$$a_n = a_1 \cdot r^{n-1}$$

Common Ratio or "The pattern"

1st Term

Let's examine a previous example

Generate the explicit formula for the following sequence:

$$1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81, \ldots$$

1st Term

What is our first term? $a_1 = 1$

What is the value of the common ratio (the "pattern")?

We are multiplying the previous term by 3.

So, r=3

Again, we use these values to create our sequence formula.

1. Write out our formula $a_n = a_1 \cdot r^{n-1}$
2. Plug in our values for r and a_1 $a_n = 1 \cdot 3^{n-1}$
3. Simplify $a_n = 3^{n-1}$

Generating an Arithmetic Sequence Formula

Example

Generate the formula for the following arithmetic sequence:

9, 16, 23, 30, 37, ...

1. The first term, a_1, is 9. So $a_1 = 9$

2. The difference between each term (or "the pattern" we observe") is +7. So $d = 7$

Now we plug those values into our formula for an ARITHMETIC Sequence.

$$a_n = a_1 + d(n-1)$$

$$a_n = 9 + 7(n-1)$$

Generating an Arithmetic Sequence Formula

Example

What is the explicit formula for the given arithmetic sequence? Afterwards find the value of the 10th term in the sequence.

-12 -12 -12 -12

101, 89, 77, 65, 53, ...

$a_1 = 101$

$d = -12$

$a_n = a_1 + d(n-1)$

$a_n = 101 - 12(n-1)$

$a_{10} = 101 - 12(10-1)$

$= 101 - 12(9)$

$= 101 - 108$

$\boxed{a_{10} = -7}$

Generating an Geometric Sequence Formula

Example

Generate the formula for the following geometric sequence:

3, 12, 48, 192, ...

1. The first term, a_1 , is 3. So $a_1 = 3$
2. The common ratio between each term (or "the pattern" we observe) is multiply by 4. So $r = 4$

Now we plug those values into our formula for a GEOMETRIC Sequence.

$$a_n = a_1 \cdot r^{n-1}$$

$$\boxed{a_n = 3 \cdot 4^{n-1}}$$

This is our formula for this sequence

Generating an Geometric Sequence Formula

Example:

Find the explicit formula for the given geometric sequence, then determine the value of the 12th term.

$$1024 \xrightarrow{\cdot\frac{1}{2}} 512 \xrightarrow{\cdot\frac{1}{2}} 256 \xrightarrow{\cdot\frac{1}{2}} 128, \ldots$$

1st Term $a_1 = 1024$

$r = \frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1024 \cdot \left(\frac{1}{2}\right)^{n-1}$

Now we need to find the 12th term using this formula we just created. We do this by substituting n=12.

1. Substitute 12 for n. $$a_{12} = 1024 \cdot \left(\frac{1}{2}\right)^{12-1}$$

2. Simplify the expression in the exponent. $$a_{12} = 1024 \cdot \left(\frac{1}{2}\right)^{11}$$

3. Distribute the exponent to the numerator and denominator $$a_{12} = 1024 \cdot \left(\frac{1^{11}}{2^{11}}\right)$$

4. Evaluate 1^{11} and 2^{11} $$a_{12} = 1024 \cdot \left(\frac{1}{2048}\right)$$

5. Multiply across $$a_{12} = \frac{1024}{2048}$$

6. Simplify the fraction $$a_{12} = \frac{1}{2}$$

Homework:

Module 3

Page S.7, #1-3

Page S.8, #6, 7, 8 (a and b for all)

Done on a separate sheet of paper