

## Module 3: Lesson 1 &amp; Lesson 3

# Introduction to Sequences

**Lesson Summary**

Think of a sequence as an ordered list of elements. Give an explicit formula to define the pattern of the sequence. Unless specified otherwise, find the first term by substituting 1 into the formula.

Two types of sequences were studied:

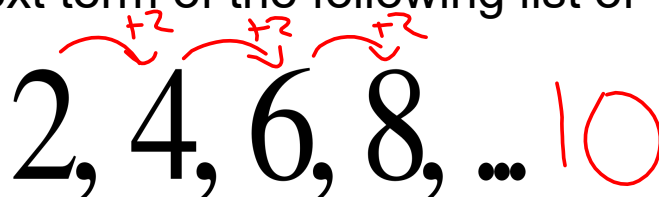
**ARITHMETIC SEQUENCE:** A sequence is called *arithmetic* if there is a real number  $d$  such that each term in the sequence is the sum of the previous term and  $d$ .

**GEOMETRIC SEQUENCE:** A sequence is called *geometric* if there is a real number  $r$  such that each term in the sequence is a product of the previous term and  $r$ .

## Opening Exercise:

What is the next term of the following list of numbers?

2, 4, 6, 8, ... 10



What pattern does the list of numbers seem to follow?

We add two to the previous term

## What is a sequence?

A sequence is an ordered list of elements, such as numbers, that follows a particular pattern. Each entry in a sequence is usually referred to as an "element" or a "term" of the sequence.

A sequence is represented by a function where  $n$  is our variable and where our domain is the positive integers.

## Notation

While sequences can be represented through function notation, we more commonly represent it through *subscript notation*.

• function variable, input, output is  $n$

$$f(n) = a_n$$

Sequence  
Index

Note: The Index tells us where we are in the sequence. For example, if  $n=1$  we are looking for the value of the first term. If  $n=10$  we are looking for the value of the 10th term.

This is read as "a sub n" where  $n$  acts like our input for a function.

We use subscript notation to indicate that we are working with a sequence, not just a general function.

Example:

Find the next three terms of the following sequence:

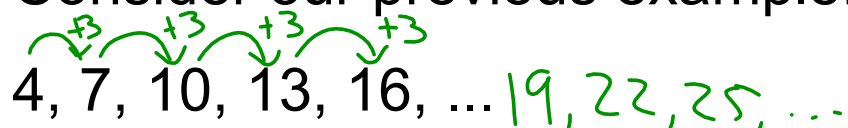
4, 7, 10, 13, 16, ... 19, 22, 25  
① ② ③ ④ ⑤ ... ⑥ ⑦ ⑧

20<sup>th</sup> term? 61

## Finding patterns to sequences

When we define sequences we find the "pattern" just like finding a "rule" to a function.

Consider our previous example:

  
4, 7, 10, 13, 16, ... 19, 22, 25, ...


What was our pattern that we observed? That is, what did we do to the previous term of our sequence in order to get to the next term?

**We added 3 to every previous term**

## Different kinds of sequences

Arithmetic sequence: A sequence in which the terms increase or decrease by a fixed amount.

Ex: 1, 3, 5, 7, 9, ...




The diagram shows the sequence 1, 3, 5, 7, 9, ... with green arrows pointing from each term to the next. Each arrow is labeled with '+2', indicating that 2 is added to each term to get the next term.

In this example, we are adding 2 to the previous term.

Geometric sequence: A sequence in which the terms increase or decrease by a fixed non-zero MULTIPLE (this is called the "common ratio").

Ex: 1, 3, 9, 27, 81, ... 243



The diagram shows the sequence 1, 3, 9, 27, 81, ... 243 with green arrows pointing from each term to the next. Each arrow is labeled with 'x3', indicating that each term is multiplied by 3 to get the next term.

In this example we are multiplying the previous term by 3 in order to get the value of the next term.

Example:

Find the following terms of the given arithmetic sequence:

$a_9 = 19$   
 $a_{10} = 21$   
 $a_{11} = 23$   
 $a_{12} = 25$

$a_1 = 3$   
 $a_4 = 9$   
 $a_8 = 17$   
 $a_{12} = 25$

1st term

3, 5, 7, 9, 11, 13, 15, 17 ...

+2 +2 +2 +2 +2 +2 +2



Example:

Find the following terms of the given geometric sequence:

$$\overset{\textcircled{1}}{4}, \overset{\textcircled{2}}{8}, \overset{\textcircled{3}}{16}, \overset{\textcircled{4}}{32}, \overset{\textcircled{5}}{64}, \overset{\textcircled{6}}{128}, 256, \dots$$

*(Red arrows labeled 'x2' point from 4 to 8, 8 to 16, and 16 to 32.)*

$$\textcircled{a_1} = 4$$

*1<sup>st</sup> Term*

$$\textcircled{a_2} = 8$$

*2<sup>nd</sup> term*

$$\textcircled{a_6} = 128$$

$$\textcircled{a_8} = 512$$

Example:

Find the 1st, 5th, and 12th terms of the following arithmetic sequence:

$$a_1 = 3$$

$$a_5 = 11$$

$$a_{12} = 25$$

$$a_n = 3 + 2(n - 1)$$

$$\begin{aligned} a_1 &= 3 + 2(1 - 1) \\ &= 3 + 2(0) \\ &= 3 + 0 \\ &= 3 \end{aligned}$$

$$\begin{aligned} a_5 &= 3 + 2(5 - 1) \\ &= 3 + 2(4) \\ &= 3 + 8 \\ &= 11 \end{aligned}$$

$$\begin{aligned} a_{12} &= 3 + 2(12 - 1) \\ &= 3 + 2(11) \\ &= 3 + 22 \\ &= 25 \end{aligned}$$

Example:

Find the 1st, 3rd, and 4th terms of the following geometric sequence:

$$a_n = 3^{n-1}$$

$$a_1 = 1$$

$$\begin{aligned} a_1 &= 3^{1-1} \\ &= 3^0 \\ &= 1 \end{aligned}$$

$$a_3 = 9$$

$$\begin{aligned} a_3 &= 3^{3-1} \\ &= 3^2 \\ &= 9 \end{aligned}$$

$$a_4 = 27$$

$$\begin{aligned} a_4 &= 3^{4-1} \\ &= 3^3 \\ &= 27 \end{aligned}$$

## Explicit formula for an **ARITHMETIC** sequence

(Use if adding or subtracting terms)

$$a_n = a_1 + d(n-1)$$

$a_1$  is the 1<sup>st</sup> Term  
 $d$  is "The difference" or The pattern

Let's take a look at a previous example

Generate the explicit formula for the following sequence:

$4, 7, 10, 13, 16, \dots$   
 (4) is the 1<sup>st</sup> Term  
 +3, +3, +3, +3

What is our first term?  $a_1 = 4$

What is the "pattern" of our sequence?

We are adding 3 to the previous terms.

So,  $d=3$

Now we will use these values to create our explicit formula:

1. Write out the formula that we're using

2. Plug in our values for  $d$  and  $a_1$

$$a_n = a_1 + d(n-1)$$

$$a_n = 4 + 3(n-1)$$

## Explicit formula for a GEOMETRIC sequence

(Use if multiplying or dividing terms)

$$a_n = a_1 \cdot r^{n-1}$$

$a_1$  is the 1<sup>st</sup> Term  
 $r$  is the Common Ratio or "The pattern"

Let's examine a previous example

Generate the explicit formula for the following sequence:

$1, 3, 9, 27, 81, \dots$   
 (1 is circled)  
 Arrows labeled  $\times 3$  connect 1 to 3, 3 to 9, 9 to 27, and 27 to 81.  
 1 is labeled 1<sup>st</sup> Term.

What is our first term?  $a_1 = 1$

What is the value of the common ratio (the "pattern")?

We are multiplying the previous term by 3.

So,  $r=3$

Again, we use these values to create our sequence formula.

1. Write out our formula  $a_n = a_1 \cdot r^{n-1}$
2. Plug in our values for  $r$  and  $a_1$   $a_n = 1 \cdot 3^{n-1}$
3. Simplify  $a_n = 3^{n-1}$

## Generating an Arithmetic Sequence Formula

### Example

Generate the formula for the following arithmetic sequence:

9, 16, 23, 30, 37, ...

1. The first term,  $a_1$ , is 9. So  $a_1 = 9$

2. The difference between each term (or "the pattern" we observe") is +7. So  $d = 7$

Now we plug those values into our formula for an ARITHMETIC Sequence.

$$a_n = a_1 + d(n - 1)$$

$$a_n = 9 + 7(n - 1)$$

## Generating an Arithmetic Sequence Formula

### Example

What is the explicit formula for the given arithmetic sequence? Afterwards find the value of the 10th term in the sequence.

101, 89, 77, 65, 53, ...

$$a_1 = 101$$

$$d = -12$$

$$a_n = a_1 + d(n-1)$$

$$a_n = 101 - 12(n-1)$$

$$a_{10} = 101 - 12(10-1)$$

$$= 101 - 12(9)$$

$$= 101 - 108$$

$$a_{10} = -7$$

## Generating an Geometric Sequence Formula

### Example

Generate the formula for the following geometric sequence:

3, 12, 48, 192, ...

1. The first term,  $a_1$ , is 3. So  $a_1 = 3$

2. The common ratio between each term (or "the pattern" we observe) is multiply by 4.

So  $r = 4$

Now we plug those values into our formula for a GEOMETRIC Sequence.

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = 3 \cdot 4^{n-1}$$

This is our formula for this sequence



## Generating an Geometric Sequence Formula

Example:

Find the explicit formula for the given geometric sequence, then determine the value of the 12th term.

$1024, 512, 256, 128, \dots$   
 1<sup>st</sup> Term  $a_1 = 1024$   
 $r = \frac{1}{2}$   
 $a_n = a_1 \cdot r^{n-1}$   
 $a_n = 1024 \cdot \left(\frac{1}{2}\right)^{n-1}$

Now we need to find the 12th term using this formula we just created. We do this by substituting  $n=12$ .

1. Substitute 12 for  $n$ .

$$a_{12} = 1024 \cdot \left(\frac{1}{2}\right)^{12-1}$$

2. Simplify the expression in the exponent.

$$a_{12} = 1024 \cdot \left(\frac{1}{2}\right)^{11}$$

3. Distribute the exponent to the numerator and denominator

$$a_{12} = 1024 \cdot \left(\frac{1^{11}}{2^{11}}\right)$$

4. Evaluate  $1^{11}$  and  $2^{11}$

$$a_{12} = 1024 \cdot \left(\frac{1}{2048}\right)$$

5. Multiply across

$$a_{12} = \frac{1024}{2048}$$

6. Simplify the fraction

$$a_{12} = \frac{1}{2}$$

Homework:

Module 3

Page S.7, #1-3

Page S.8, #6, 7, 8 (a and b for all)

Done on a separate sheet of paper