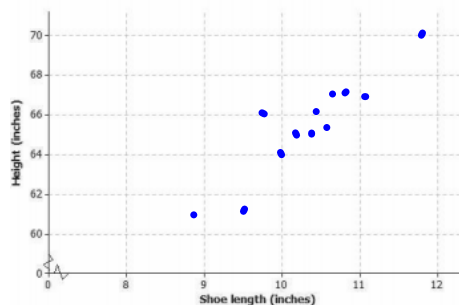


Warm-Up (10/10/17)

The following data set was collected in order to determine if the shoe length of a woman (in inches) and their height are correlated.

x (Shoe Length of Women)	y (Height of Women)
8.9	61
9.6	61
9.8	66
10.0	64
10.2	64
10.4	65
10.6	65
10.6	67
10.5	66
10.8	67
11.0	67
11.8	70

1. Construct a scatter plot of this data.



2. The equation for the line of best fit is $y = 32.68 + 3.15x$. Use this to calculate the residual for when $x = 10.8$

$$\text{Residual} = \text{Actual } y\text{-value} - \text{Predicted } y\text{-value}$$

↓
y-value given to us, usually from our table.

↓
Our y-value when we plug X into our equation for our "Line of best fit"

$$\text{Actual: } y = 67$$

$$\begin{aligned} \text{Prediction: } y &= 32.68 + 3.15(10.8) \\ &= 32.68 + 34.02 \\ &= 66.7 \end{aligned}$$

$$\begin{aligned} \text{Residual} &= 67 - 66.7 \\ &= 0.3 \end{aligned}$$

Module 2: Lesson 15

Interpreting Residuals from a Line

Lesson Summary

- When a least squares line is used to calculate a predicted value, the prediction error can be measured by
$$\text{residual} = \text{actual } y\text{-value} - \text{predicted } y\text{-value}.$$
- On the graph, the residuals are the vertical distances of the points from the least squares line.
- The residuals give us an idea how close a prediction might be when the least squares line is used to make a prediction for a value that is not included in the data set.

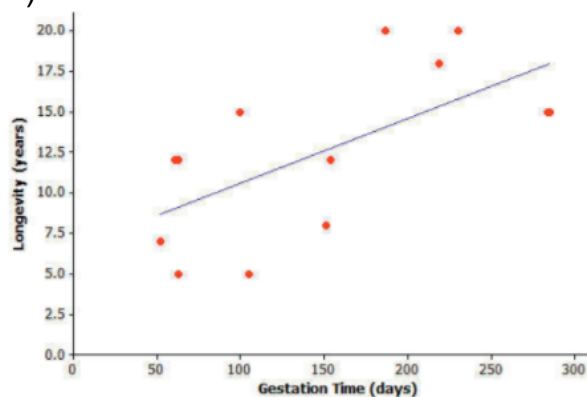
The gestation time for an animal is the typical duration between conception and birth. The longevity of an animal is the typical lifespan for that animal. The gestation times (in days) and longevity (in years) for 13 types of animals are shown in the table below.

Animal	Gestation Time (days)	Longevity (years)
Baboon	187	20
Black Bear	219	18
Beaver	105	5
Bison	285	15
Cat	63	12
Chimpanzee	230	20
Cow	284	15
Dog	61	12
Fox (Red)	52	7
Goat	151	8
Lion	100	15
Sheep	154	12
Wolf	63	5

Data Source: Core Math Tools, <http://nctm.org>

1. The equation of the least squares line ("line of best fit") is $y = 6.642 + 0.03974x$

Remember that x represents gestation time (in days) and y represents longevity (in years).



The gestation time for an animal is the typical duration between conception and birth. The longevity of an animal is the typical lifespan for that animal. The gestation times (in days) and longevity (in years) for 13 types of animals are shown in the table below.

Animal	Gestation Time (days)	Longevity (years)
Baboon	187	20
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Data Source: Core Math Tools, <http://nctm.org>

$$y = 6.642 + 0.03974x$$

2. Suppose that a particular type of animal has a gestation time of 200 days. What is the approximate longevity of this particular animal?

In other words, what is the y-value when $x = 200$?

$$y = 6.642 + 0.03974(200)$$

$$y = 14.59$$

The longevity of an animal with a gestation time of 200 days is approximately 14.59 years.

3. Would the value you predicted in Exercise 2 necessarily be the exact value for the longevity of that type of animal? Could the actual longevity of that type of animal be longer than predicted? Could it be shorter?

No, because there are many other factors that we should consider when we predict an animal's longevity. We should also note that the number we got is just a PREDICTION we received from our least squares equation.

The gestation time for an animal is the typical duration between conception and birth. The longevity of an animal is the typical lifespan for that animal. The gestation times (in days) and longevities (in years) for 13 types of animals are shown in the table below.

$$y = 6.642 + 0.03974x$$

Animal	Gestation Time (days)	Longevity (years)
Baboon	187	20
Black Bear	219	18
Beaver	105	5
Bison	285	15
Cat	63	12
Chimpanzee	230	20
Cow	284	15
Dog	61	12
Fox (Red)	52	7
Goat	151	8
Lion	100	15
Sheep	154	12
Wolf	63	5

4. Calculate the residual of a lion, who has a gestation time of 100 days.

Actual value: 15

Prediction: 10.6

$$15 - 10.6 = 4.4$$

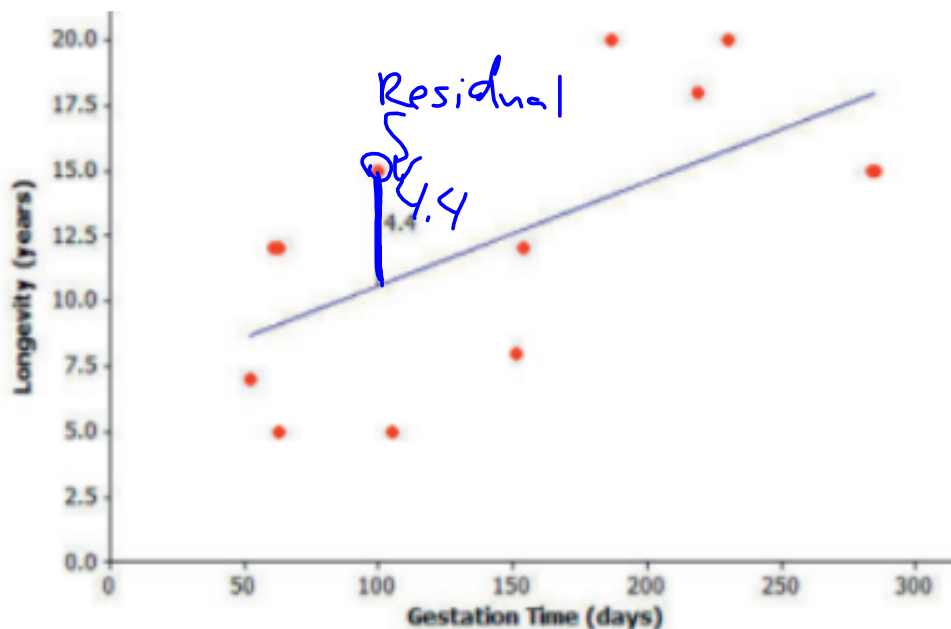
Residual value
or
Prediction error

Data Source: Core Math Tools, <http://nctm.org>

Recall:

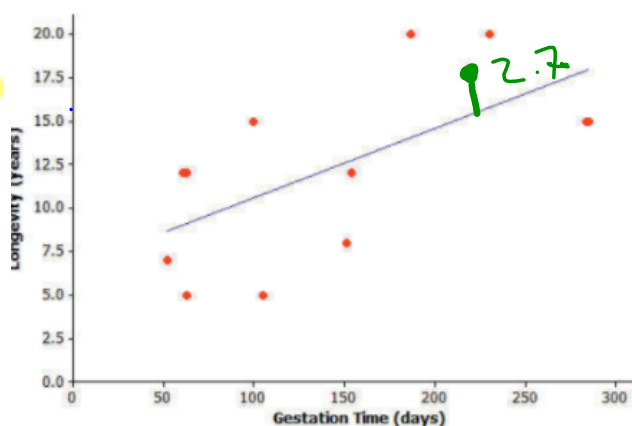
$$\text{Residual} = \text{Actual } y\text{-value} - \text{Predicted } y\text{-value}$$

You can show the prediction error of 4.4 years on the graph like this:



The gestation time for an animal is the typical duration between conception and birth. The longevity of an animal is the typical lifespan for that animal. The gestation times (in days) and longevity (in years) for 13 types of animals are shown in the table below.

Animal	Gestation Time (days)	Longevity (years)
Baboon	187	20
Black Bear	219	18
Beaver	105	5
Bison	285	15
Cat	63	12
Chimpanzee	230	20
Cow	284	15
Dog	61	12
Fox (Red)	52	7
Goat	151	8
Lion	100	15
Sheep	154	12
Wolf	63	5



Data Source: Core Math Tools, <http://nctm.org>

5. Let's continue to think about the gestation times and longevity of animals. Let's specifically investigate how accurately the least squares line predicted the longevity of the black bear.

a. What is the gestation time for the black bear?

219 days

b. Look at the graph. Roughly what does the least squares line predict for the longevity of the black bear?

About 15 years
(Just an estimation from looking at the graph)

c. Use the gestation time from part (a) and the least squares line $y = 6.642 + 0.03974x$ to predict the black bear's longevity. Round your answer to the nearest tenth.

$$y = 6.642 + 0.03974(219)$$

$$y = 15.3 \text{ years}$$

d. What is the actual longevity of the black bear?

18 years

e. How much do you have to add to the predicted value to get the actual longevity of the black bear?

$$18 - 15.3 = 2.7$$

f. Show your answer to part (e) on the graph as a vertical line segment.

(Shown on graph above)

Below is the same table we've seen, but now it has a column for the residuals for the longevity of each animal.

Animal	Gestation Time (days)	Longevity (years)	Residual (years)
Baboon	187	20	5.9
Black Bear	219	18	2.7
Beaver	105	5	-5.8
Bison	285	15	-3.0
Cat	63	12	2.9
Chimpanzee	230	20	4.2
Cow	284	15	-2.9
Dog	61	12	2.9
Fox (Red)	52	7	-1.7
Goat	151	8	-4.6
Lion	100	15	4.4
Sheep	154	12	-0.8
Wolf	63	5	-4.1

These residuals show that the actual longevity of an animal should be within six years of the longevity predicted by the least squares line.

Suppose you selected a type of animal that is not included in the original data set, and the gestation time for this type of animal is 270 days. Substituting $x = 270$ into the equation of the least squares line you get

$$y = 6.642 + 0.03974(270) \\ = 17.4.$$

The predicted longevity of this animal is 17.4 years.

Think about what the *actual* longevity of this type of animal might be.

7. Could it be 30 years? How about 5 years?

It is unlikely that the animal could live as long as 30 years or as short as 5 years. This is because our residual data would be 12 and -13, which is very far off from our given residual data set.

8. Judging by the size of the residuals in our table, what kind of values do you think would be reasonable for the longevity of this type of animal?

Since our residual data seems to be between 6 and -6 we can conclude that the actual longevity of this type of animal could realistically be between 11.4 years and 23.4 years.

Continue to think about the gestation times and longevities of animals. The gestation time for the type of animal called the ocelot is known to be 85 days.



The least Squares line
Predicts the following about
an ocelot's longevity:

$$\begin{aligned} Y &= 6.642 + 0.03974(85) \\ &= 6.642 + 3.3779 \\ &= 10.019 \\ &= 10.0 \text{ (rounded up)} \end{aligned}$$



9. Based on the residuals in Example 3, would you be surprised to find that the longevity of the ocelot was 2 years? Why or why not? What might be a sensible range of values for the actual longevity of the ocelot?

Yes, because it is unlikely that an ocelot will live for as little as 2 years based on our residual data. A good range for its longevity would be between 4 to 16 years.

10. We know that the actual longevity of the ocelot is 9 years. What is the residual for the ocelot?

$$\begin{aligned} \text{Residual} &= \text{Actual value} - \text{Predicted value} \\ &= 9 - 10 \\ &= -1 \end{aligned}$$