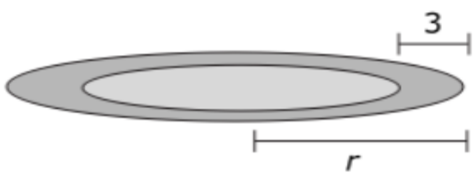
Warm Up (2/15/18) PARCC Practice

A circular pool of water is shrinking as it drains. The diagram shows the shrinkage.



A formula for the area, *A*, of the circular pool is given by the equation $A = \pi(r-3)^2$.

Solve for r

Which is a formula for *r*?

A. $r = \sqrt{\frac{A}{\pi}} - 3$

B. $r = \frac{\sqrt{A}}{\pi} + 3$

C. $r = \sqrt{\frac{A}{\pi}} + 3$

D. $r = \sqrt{\frac{A}{\pi} - 3}$

$$\frac{A}{\pi} = \frac{\cancel{\pi}(r-3)^2}{\cancel{\pi}}$$

$$\sqrt{\frac{A}{\pi}} = \sqrt{(r-3)^2}$$

$$\sqrt{\frac{A}{\pi}} = r - 3$$

$$+3 \qquad +3$$

$$\sqrt{\frac{A}{\pi}} + 3 = r$$

Module 4: Lesson 9

Graphing Quadratic Equations in Factored Form

Pg. 55

Lesson Summary

- When we have a quadratic function in factored form, we can find its x-intercepts, y-intercept, axis of symmetry, and vertex.
- For any quadratic equation, the roots are the solution(s) where $y = 0$, and these solutions correspond to the points where the graph of the equation crosses the x-axis.
- A quadratic equation can be written in the form $y = a(x - m)(x - n)$, where m and n are the roots of the function. Since the x-value of the vertex is the average of the x-values of the two roots, we can substitute that value back into the equation to find the y-value of the vertex. If we set $x = 0$, we can find the y-intercept.

Solve the following equation.

$$x^2 + 6x - 40 = 0$$

① Factor ✓
② Set factors equal to zero ✓
③ Solve each equation ✓

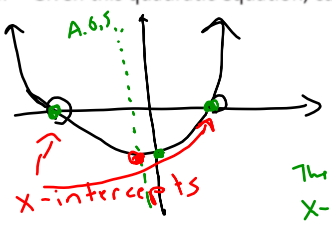
-40, 10, -4, 6

$(x+10)(x-4) = 0$

$x + 10 = 0$
$-10 \quad -10$
$\boxed{x = -10}$

$x - 4 = 0$
$+4 \quad +4$
$\boxed{x = 4}$

a. Given this quadratic equation, can you find the point(s) where the graph crosses the x-axis?



① $(-10, 0)$
② $(4, 0)$

The points where the equation crosses the x-axis are the same x-values we solved for previously.

b. In the last lesson, we learned about the symmetrical nature of the graph of a quadratic function. How can we use that information to find the vertex for the graph?

$y = x^2 + 6x - 40$

Average of the x-values of x-intercepts gives the Axis of Symmetry.

$\frac{-10+4}{2} = \frac{-6}{2} = -3$

$x = -3$
$y = (-3)^2 + 6(-3) - 40$
$= 9 - 18 - 40 = -9 - 40 = \boxed{-49}$

③ $(-3, -49)$

c. How could we find the y-intercept (where the graph crosses the y-axis and where $x = 0$)

$y = x^2 + 6x \boxed{-40}$

$y = 0^2 + 6(0) - 40$
$y = 0 + 0 - 40 = -40$

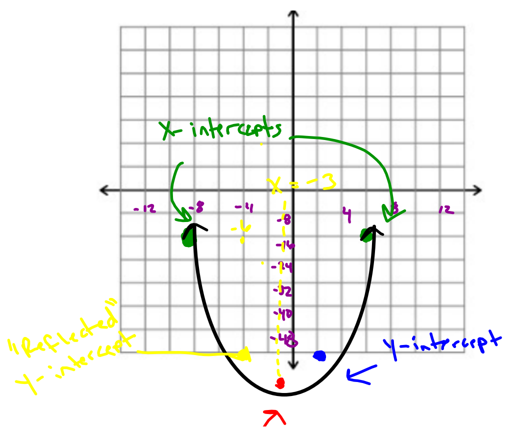
④ $(0, -40)$

d. What else can we say about the graph based on our knowledge of the symmetrical nature of the graph of a quadratic function? Can we determine the coordinates of any other points?

1 more point is needed. Usually we can choose the point that is the "reflected" point of the y-intercept.

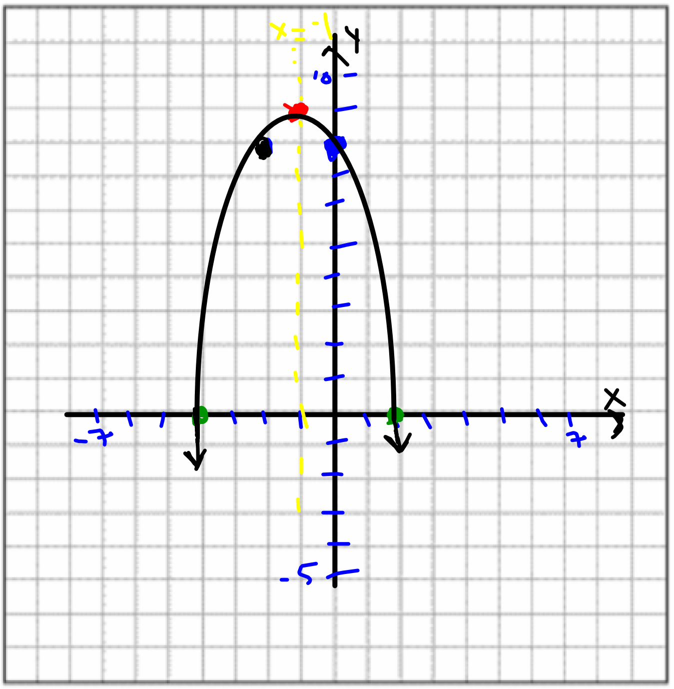
e. Plot the points you know for this equation on graph paper, and connect them to show the graph of the equation.

- x-intercepts: $(-10, 0)$ $(4, 0)$
- Vertex $(-3, -49)$
- Y-intercept $(0, -40)$
- One more point $(-6, -40)$



Graph the following by identifying the key features of the graph.

$$f(x) = -(x+4)(x-2)$$



• x-intercepts:

$x+4=0$ or $x-2=0$

$-4 \; -4$ $\qquad +2 \; +2$

$\boxed{x=-4}$ or $\boxed{x=2}$

$(-4, 0)$ or $(2, 0)$

• Vertex

(Take the average of our 2 x-intercepts)

$$x = \frac{-4+2}{2} = \frac{-2}{2} = -1$$

$\Rightarrow$ $x=-1$ (Plug this x-value into our original equation to get our y-value)

$f(x) = -(x+4)(x-2)$

$f(-1) = -[(-1)+4]((-1)-2)$

$= -(3)(-3)$

$= -(-9) = 9$

The point for our vertex is $(-1, 9)$

• y-intercept

(Substitute $x=0$ into the original equation to find any y-intercept)

$f(0) = -(0+4)(0-2)$

$= -(4)(-2)$

$= -(-8)$

$= 8$

So our y-intercept is at the point $(0, 8)$.

• One more point

$(-2, 8)$ is the point that is reflected across the axis of symmetry that "mirrors" the y-intercept.

Both are 1 unit away from the axis of symmetry so both share the same y-value.

Graph the following by identifying the key features of the graph $g(x) = x^2 - 5x - 24$

① x-intercepts

$g(x) = x^2 - 5x - 24$

-24, 3, -8, -5

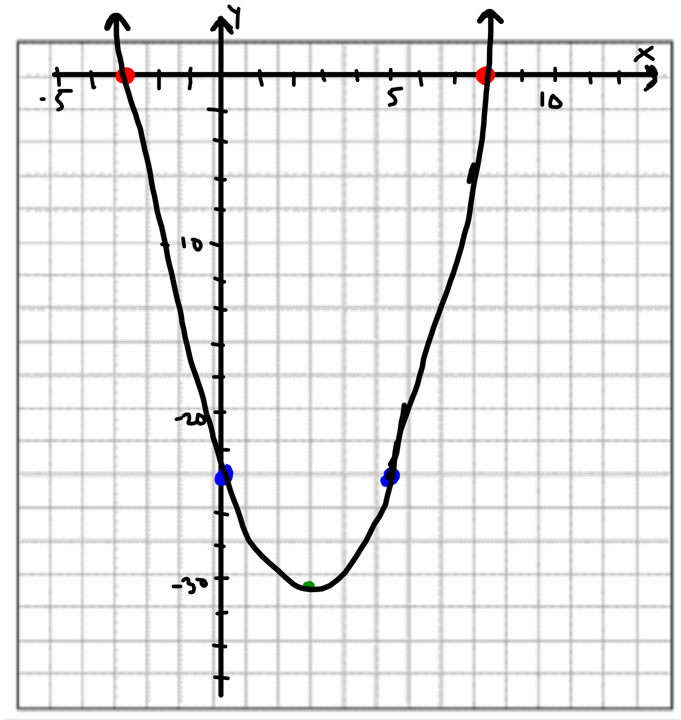
$g(x) = (x+3)(x-8)$

$x+3=0$ or $x-8=0$

$-3 \quad -3$ $\quad +8 \quad +8$

$x=-3$ or $x=8$

$\boxed{(-3,0) \text{ or } (8,0)}$



② Vertex

$x = \frac{-3+8}{2} = \frac{5}{2} = 2.5$

$f(2.5) = (2.5)^2 - 5(2.5) - 24$

$= 6.25 - 12.5 - 24$

$= -30.25$

So, vertex is the point $(2.5, -30.25)$

③ y-intercept

$f(x) = x^2 - 5x - 24$

$f(0) = (0)^2 - 5(0) - 24$

$= 0 - 0 - 24$

$= -24$

y-intercept is $(0, -24)$

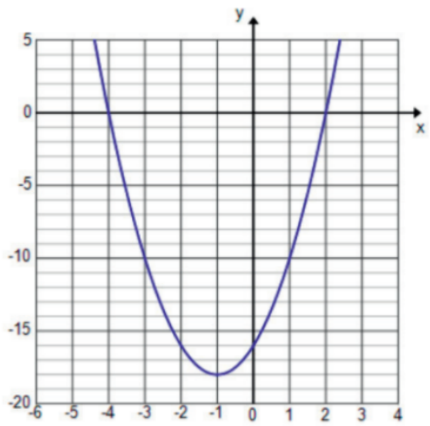
④ One more point

$(5, -24)$

Creating a Quadratic Equation from the Graph

Example 2

Consider the graph of the quadratic function shown below with x-intercepts -4 and 2.



a. Write a formula for a possible quadratic function, in factored form, that the graph represents using a as a constant factor.

$f(x) = a(x+4)(x-2)$

b. The y −intercept of the graph is -16. Use the y-intercept to adjust your function by finding the constant factor a.

$16 = a(0+4)(0-2)$

$\frac{16}{-8} = \frac{a(-8)}{-8}$

$a = -2$

So our equation is $f(x) = -2(x+4)(x-2)$