Warm Up (11/16/17)

Find the first 6 terms of the sequence defined by the recursive formula $a_n = a_{n-1} + 7$ where $a_1 = 3$ and $n \geq 2$.

$a_1 = 3$

$a_2 = a_{2-1} + 7 = a_1 + 7 = 3 + 7 = 10$

$a_3 = a_{3-1} + 7 = a_2 + 7 = 10 + 7 = 17$

$a_4 = 24$

$a_5 = 31$

$a_6 = 38$

Module 3: Lesson 5

The Power of Exponential Growth

Two equipment rental companies have different penalty policies for returning a piece of equipment late.

Company 1: On day 1, the penalty is $5. On day 2, the penalty is $10. On day 3, the penalty is $15. On day 4, the penalty is $20, and so on, increasing by $5 each day the equipment is late.

Company 2: On day 1, the penalty is $0.01. On day 2, the penalty is $0.02. On day 3, the penalty is $0.04. On day 4, the penalty is $0.08, and so on, doubling in amount each additional day late.

Jim rented a digger from Company 2 because he thought it had the better late return policy. The job he was doing with the digger took longer than he expected, but it did not concern him because the late penalty seemed so reasonable. When he returned the digger 15 days late, he was shocked by the penalty fee. What did he pay, and what would he have paid if he had used Company 1 instead?

Arithmetic

+5 +5 +5

Company 1	
Day	Penalty
1	$5
2	$10
3	$15
4	$20
5	$25
6	$30
7	$35
8	$40
9	$45
10	$50
11	$55
12	$60
13	$65
14	$70
15	$75

Geometric

x2 x2 x2

Company 2	
Day	Penalty
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64
8	$1.28
9	$2.56
10	$5.12
11	$10.24
12	$20.48
13	$40.96
14	$81.92
15	$163.84

a. Which company has a greater 15-day late charge?

Company 2 .

b. Describe how the amount of the late charge changes from any given day to the next successive day in both Companies 1 and 2.

Company 1
Add $5 Every late day

Company 2
Doubles Every late day

c. How much would the late charge have been after 20 days under Company 2?

$5,242.88

Exponential Functions

Exponential functions are typically seen in the following form:

$$f(x) = a \cdot b^x$$

- *a* is called the "scale factor". Typically this will be set to 1.
- *b* is called the "base"
- The input, *x*, will indicate how many times the base will be multiplied by itself.
- By definition, $b^0 = 1$
- For negative exponents we evaluate the reciprocal. For example:

① $2^{-1} = \frac{1}{2^{1}} = \frac{1}{2}$

② $3^{-2} = \frac{1}{3^{2}} = \frac{1}{9}$

③ $\frac{1}{4^{-2}} = \frac{4^{2}}{1} = 16$

Note: When we evaluate the reciprocal, the number in the exponent changes signs.

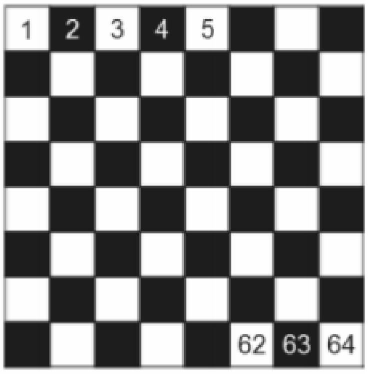
Example 1

Folklore suggests that when the creator of the game of chess showed his invention to the country's ruler, the ruler was highly impressed. He was so impressed, he told the inventor to name a prize of his choice. The inventor, being rather clever, said he would take a grain of rice on the first square of the chessboard, two grains of rice on the second square of the chessboard, four on the third square, eight on the fourth square, and so on, doubling the number of grains of rice for each successive square. The ruler was surprised, even a little offended, at such a modest prize, but he ordered his treasurer to count out the rice.

The treasurer took more than a week to count the rice in the ruler's store, only to notify the ruler that it would take more rice than was available in the entire kingdom. Shortly thereafter, as the story goes, the inventor became the new king.

b. Imagine the treasurer counting the needed rice for each of the 64 squares. We know that the first square is assigned a single grain of rice, and each successive square is double the number of grains of rice of the previous square. The following table lists the first five assignments of grains of rice to squares on the board. How can we represent the grains of rice as exponential expressions?

Square #	Grains of Rice	Exponential Expression
1	1	2^0
2	2	2^1
3	4	2^2
4	8	2^3
5	16	2^4



c. Write the exponential expression that describes how much rice is assigned to each of the last three squares of the board.

Square #	Exponential Expression
62	2^{61}
63	2^{62}
64	2^{63}

Using graphing we will compare the difference between the following functions in order to compare the growth each one exhibits.

$f(x) = 2x$

−2	−4
−1	−2
0	0
1	2
2	4
3	6

$g(x) = 2^x$

−2	.25 or $\frac{1}{4}$
−1	.5 or $\frac{1}{2}$
0	1
1	2
2	4
3	8

÷2

÷2

÷2

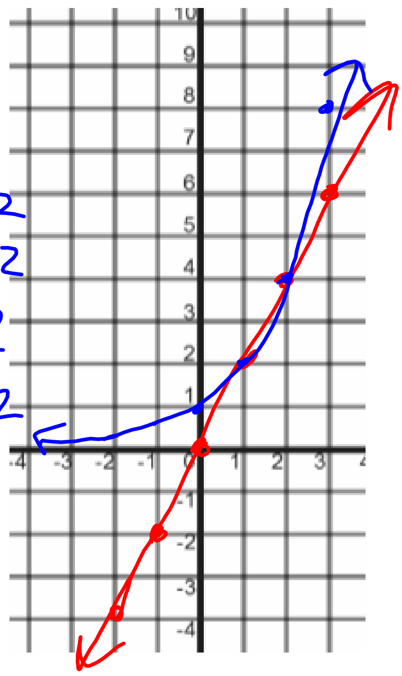
÷2

$g(3) = 2^3 = 2 \cdot 2 \cdot 2$

$g(2) = 2^2 = 2 \cdot 2$

$g(1) = 2^1 = 2$

$g(0) = 2^0 = 1$



Rates of Change

"Slope"

Rate of change describes how one variable changes in relation to another. $\frac{\text{Change in } y}{\text{Change in } x}$

Linear functions have a constant rate of change. $y = mx + b$

Quadratic functions do not have a constant rate of change.

Exponential functions do not have a constant rate of change, but instead will either rapidly increase or decrease. This is called exponential GROWTH or DECAY.

A typical thickness of toilet paper is 0.001 inch. This seems pretty thin, right? Let's see what happens when we start folding toilet paper.

a. How thick is the stack of toilet paper after 1 fold? After 2 folds? After 5 folds?

1 fold: 0.002 in
2 folds: 0.004 in
3 folds: 0.008 in
4 folds: 0.016 in
5 folds: 0.032 in

b. Write an explicit formula for the sequence that models the thickness of the folded toilet paper after n folds.

Since this is geometric, we use the formula $a_n = a_1 r^{n-1}$

$a_n = 0.002(2^{n-1})$

c. After how many folds does the stack of folded toilet paper pass the 1-foot mark?

14 folds: $a_{14} = 0.002(2^{14-1}) = .002(2^{13}) = .002(8192)$
$= 16.384$ in

d. The moon is about 240,000 miles from Earth. Compare the thickness of the toilet paper folded 50 times to the distance from Earth.

$a_{50} = .002(2^{50-1}) = .002(2^{49})$

$a_{50} = .002(562,949,953,421,312)$

$= 1,125,899,906,842.624 \cancel{\text{in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in}}}$

$= 93,824,992,236.88533 \cancel{\text{ft}} \cdot \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}}$

$a_{50} = 17,769,884.89334949$ miles

The thickness of the paper is GREATER than the distance from the Earth to the Moon.