Module 4: Lesson 12

Completing the Square (Part 2)

Opening Exercise

Pg. 73

Rewrite each expression by completing the square.

a. $z^2 - 5z + 8$

$\left(z^2 - 5z + \frac{25}{4}\right) - \frac{25}{4} + 8$

$b = -5$

$\left(x + \left(\frac{b}{2}\right)\right)^2$

$\frac{b}{2} = -\frac{5}{2}$

$\left(\frac{b}{2}\right)^2 = \frac{25}{4}$

$\left(z - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{8}{1}\left(\frac{4}{4}\right)$

$-\frac{25}{4} + \frac{32}{4}$

$\frac{7}{4}$

$\left(z - \frac{5}{2}\right)^2 + \frac{7}{4}$

or

$(z - 2.5)^2 + 1.75$

Example 1

Now complete the square for $2x^2 + 16x + 3$.

$a = 2$, factor out 2

$2\left(x^2 + 8x + \frac{3}{2}\right)$

$2\left(\left(x^2 + 8x + 16\right) - 16 + \frac{3}{2}\right)$

$b = 8$

$\frac{b}{2} = 4$

$\left(\frac{b}{2}\right)^2 = 16$

$\left(x + \left(\frac{b}{2}\right)\right)^2$

$\frac{3}{2} \cdot \frac{2}{1}$

$2\left((x+4)^2 - 16 + \frac{3}{2}\right)$

$2(x+4)^2 - 32 + 3$

$\boxed{2(x+4)^2 - 29}$

1. Identify a and divide it out.

2. Rewrite the expression with two blank spaces.

3. Put parenthesis around the first 3 terms.

4. Divide the "b" term by two and square it. This number will go in both blank spaces that we created.

5. We can now write the terms inside the parenthesis as a perfect square. It will take the form $\left(x + \left(\frac{b}{2}\right)\right)^2$

6. Distribute our coefficient back in and combine the remaining terms

Exercises Pg. 75

For Exercises 1–5, rewrite each expression by completing the square.

1. $3x^2 + 12x - 8$

$a=3$, Factor out 3

$3\left(\left(x^2+4x+\underline{4}\right)-\underline{4}-\frac{8}{3}\right)$

$b=4$ → $(x+(\frac{b}{2}))^2$

$\frac{b}{2}=2$

$(\frac{b}{2})^2=4$

$3\left((x+2)^2-4-\frac{8}{3}\right)$

$3(x+2)^2-12-8$

$\boxed{3(x+2)^2-20}$

2. $4p^2 - 12p + 13$

$a=4$, factor out 4

$4\left(p^2-3p+\frac{13}{4}\right)$

$4\left(\left(p^2-3p+\_\right)-\_+\frac{13}{4}\right)$

$4\left(\left(p^2-3p+\frac{9}{4}\right)-\frac{9}{4}+\frac{13}{4}\right)$

$4\left(\left(p-\frac{3}{2}\right)^2-\frac{9}{4}+\frac{13}{4}\right)$

$4\left(p-\frac{3}{2}\right)^2-9+13$

$\boxed{4\left(p-\frac{3}{2}\right)^2+4}$

$b=-3$

$\frac{b}{2}=-\frac{3}{2}$

$(\frac{b}{2})^2=\frac{9}{4}$

3. $\frac{1}{2}y^2 + 3y - 4$

$a=\frac{1}{2}$, factor out $\frac{1}{2}$ (* everything will double when dividing by 1/2)

$\frac{1}{2}\left(y^2+6y-8\right)$

$\frac{1}{2}\left(\left(y^2+6y+\_\right)-\_-8\right)$

$\frac{1}{2}\left(\left(y^2+6y+9\right)-9-8\right)$

$\frac{1}{2}\left(\left(y+3\right)^2-9-8\right)$

$\frac{1}{2}\left(y+3\right)^2-\frac{9}{2}-\frac{4}{1}$

$\frac{1}{2}\left(y+3\right)^2-\frac{9}{2}-\frac{8}{2}$

$\boxed{\frac{1}{2}\left(y+3\right)^2-\frac{17}{2}}$

$b=6$

$\frac{b}{2}=3$

$(\frac{b}{2})^2=9$