Module 4: Lesson 7

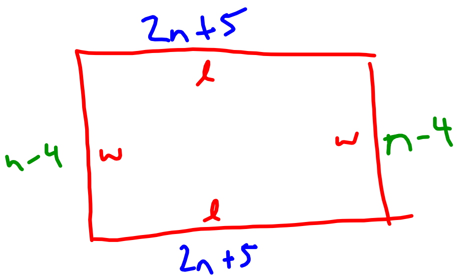
Creating and Solving Quadratic Equations in One Variable

Opening Exercise

+5 ×2 n −4 n

The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. The perimeter of the rectangle is 44 in. Sketch a diagram of this situation, and find the unknown number.

↳ P = 44



formula for perimeter:

$$2l + 2w = P$$

$$2(2n+5) + 2(n-4) = 44$$

$$4n+10 + 2n - 8 = 44$$

$$6n + 2 = 44$$

$$-2 \quad -2$$

$$6n = 42$$

$$\frac{6n}{6} = \frac{42}{6}$$

$$\boxed{n = 7}$$

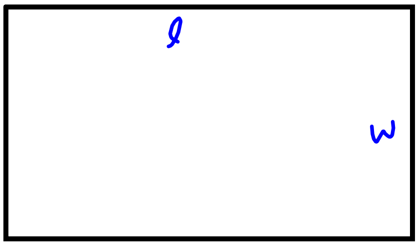
Example 1

The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. If the area of the rectangle is 15 in², find the unknown number.

$+5$ $\times 2$ n -4 n

$A=15$

$2n+5$



$n-4$

Formula for Area

$A = lw$

$(2n+5)(n-4)=15$ (FOIL or Box Method)

$2n^2-8n+5n-20=15$

$2n^2-3n-20=15$

$-15 \quad -15$

$2n^2-3n-35=0$

Factor using AC method

$2n^2-3n-35=0$

$2(-35)=-70$

$-10, 7$

-3

$2n^2-10n+7n-35=0$

$(2n^2-10n)+(7n-35)=0$

$2n(n-5)+7(n-5)=0$

$(2n+7)(n-5)=0$

Now set factors equal to zero

$2n+7=0$

$-7 \quad -7$

$2n=-7$

$\frac{2n}{2}=\frac{-7}{2}$

$\boxed{n=-\frac{7}{2}}$

$n-5=0$

$+5 \quad +5$

$\boxed{n=5}$

From our 2 answers we choose the one that makes sense in the context of the problem.

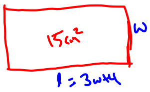
$n=-\frac{7}{2}$ will NOT make sense because we cannot have a negative area. So we discard it from our solution set.

Therefore, our only answer that is left is $\boxed{n=5.}$

Exercises

Solve the following problems. Be sure to indicate if a solution is to be rejected based on the contextual situation.

1. The length of a rectangle is 4 cm more than 3 times its width. If the area of the rectangle is 15 cm^2, find the width.



Area = $l \cdot w$

let width = w

let length = 3w+4

$15 = (3w+4)w$

$15 = 3w^2+4w$

$-15 \qquad -15$

$0 = 3w^2+4w-15$

Factor by AC method

$0 = 3w^2+4w-15$

$3\cdot(-15) = -45$; 9, −4

$0 = 3w^2+9w-5w-15$

$0 = (3w^2+9w)+(-5w-15)$

$0 = 3w(w+3)+(-5)(w+3)$

Set each factor equal to zero and solve for w

$0 = (w+3)(3w-5)$

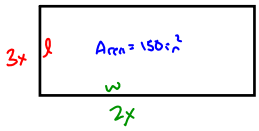
$w+3 = 0$, $-3 \quad -3$, $w = -3$

$3w-5 = 0$, $+5 \quad +5$, $\frac{3w}{3} = \frac{5}{3}$, $w = \frac{5}{3}$

Choose the answer that make sense!

$\boxed{\text{width} = \frac{5}{3} \text{ cm}}$

2. The ratio of length to width in a rectangle is 2 : 3. Find the length of the rectangle when the area is 150 in^2.



Area = $l \cdot w$

A = 150

$l = 2x$

$w = 3x$

$150 = (2x)(3x)$

$\frac{150}{6} = \frac{6x^2}{6}$

$25 = x^2$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

(only the positive answer makes sense!)

$x = 5$

$l = 2(5) = 10$ in

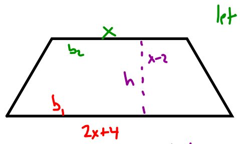
$w = 3(5) = 15$ in

$\boxed{\text{Length of the rectangle is 10 inches}}$

3. One base of a trapezoid is 4 in. more than twice the length of the second base. The height of the trapezoid is 2 in. less than the second base. If the area of the trapezoid is 4 in^2, find the dimensions of the trapezoid.
(Note: The area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)

$A = 4 \text{ in}^2$

let $b_2 = x$



$A = \frac{1}{2}(b_1 + b_2)h$

$4 = \frac{1}{2}(2x+4+x)(x-2)$

$4 = \frac{1}{2}(3x+4)(x-2)$ Multiply both sides by 2 to get rid of $\frac{1}{2}$.

$2\cdot 4 = \frac{1}{\cancel{2}}\cancel{2}(3x+4)(x-2)$

$8 = (3x+4)(x-2)$ → Multiply both binomials

$8 = 3x^2-6x+4x-8$

$8 = 3x^2-2x-8$

$-8 \qquad\quad -8$

$0 = 3x^2-2x-16$ → Factor

$0 = (3x-8)(x+2)$ → Solve each expression for x

$3x-8 = 0$, $+8 \quad +8$, $\frac{3x}{3} = \frac{8}{3}$, $x = \frac{8}{3}$ or

$x+2 = 0$, $-2 \quad -2$, or $\cancel{x = -2}$ This answer doesn't make sense in the context of the problem

Dimensions of the trapezoid

Base 1 : $\frac{28}{3}$ inches

Base 2 : $\frac{8}{3}$ inches

height : $\frac{2}{3}$ inches

$b_1 = 2x+4 = 2\left(\frac{8}{3}\right)+4 = \frac{16}{3}+4 = \frac{16}{3}+\frac{12}{3} = \frac{28}{3}$

$h = x-2 = \frac{8}{3}-2 = \frac{8}{3}-\frac{6}{3} = \frac{2}{3}$