Warm Up (1/24/18)
Factor the following quadratic expression and find the zeros:

$$x^2 - 5x - 24$$

$$(x + 3)(x - 8)$$

$$x + 3 = 0$$
$$\underline{-3 \quad -3}$$
$$\boxed{x = -3}$$

$$x - 8 = 0$$
$$\underline{+8 \quad +8}$$
$$\boxed{x = 8}$$

-24

3   8

-5

Module 4: Lessons 3 and 4

Advanced Factoring

(Leading Coefficient is greater than 1)

$$ax^2 + bx + c$$

Factor the following quadratic expression:

$$x^2 - 3x - 28$$

$$(x + 4)(x - 7)$$

-28
4 ✗ -7
-3

Guess and check

Now try this (use trial and error):

$x^2 - 3x - 28$

$$2x^2 - 6x - 56$$

1st Attempt

$$(2x + 17)(x - 3)$$
$$2x^2 - 6x + 17x - 51$$

2nd

$$(2x + 28)(x - 2)$$
$$2x^2 - 4x + 28x - 56$$

3rd

$$(2x + 18)(x - 4)$$
$$2x^2 - 8x + 18x - 72$$

Guess and Check is <u>NOT</u> the most reliable tool at our disposal!

The "Guess and Check" is a viable option, and sometimes it is our only option (for now).

Another very good example of how to use "Guess and Check" is located in Module 4, page S. 18.

Example 3

Now try factoring a quadratic expression with some negative coefficients: $3x^2 - x - 4$.

$(\_\_x \pm \_\_)(\_\_x \pm \_\_)$ [We have four number places to fill in this factor template.]

$(\_\_x \pm 1)(\_\_x \pm 4)$ [We know that ± 1 and ± 4 or ± 2 and ± 2 are the only possible factors for the constant term, -4, so we start there. Try 1 and 4 to start, and if that does not work, go back and try ± 2 and ± 2. We know that only one of the numbers can be negative to make the product negative.]

$(1x \pm 1)(3x \pm 4)$ [We know that 3 and 1 are the only factors of 3. We also know that both of these are positive (or both negative). But we do not know which positions they should take, so we will try both ways to see which will give a sum of -1.]

$(x + 1)(3x - 4)$ [Finally, we determine the two signs needed to make the final product $3x^2 - x - 4$.]

Factoring Tool #1

Factor out the Greatest Common Factor

$$2x^2 - 6x - 56$$

Identify a common factor and pull it out of the expression

$$2(x^2 - 3x - 28)$$

$$2(x - 7)(x + 4)$$

$$-28$$

$$-3$$

Factor the expression on the inside of the parenthesis.

(This still works with variables as factors too.)

Example 1

$$3x^3 - 33x^2 - 36x$$

$$3(x^3 - 11x^2 - 12x)$$

$$3x(x^2 - 11x - 12)$$

$$3x(x + 1)(x - 12)$$

$$-12$$

$$1 \quad -12$$

$$-11$$

Example 2

$$12x^2 + 36x + 24$$

$$12(x^2 + 3x + 2)$$

$$12(x + 1)(x + 2)$$

$$2$$

$$1 \quad 2$$

$$3$$

Try these Examples from Module 4

S. 23, Part a

$$2x^2 + 10x + 12$$

Factor out a 2

$$2(x^2 + 5x + 6)$$

$$2(x+2)(x+3)$$

$$\frac{6}{2 \times 3}$$

S. 21, #2

$$3x^2 + 27x + 60$$

Factor out a 3

$$3(x^2 + 9x + 20)$$

$$3(x+4)(x+9)$$

$$\frac{20}{4 \times 5}$$
$$9$$

Factoring Tool #2

Factor by Grouping

If there are more than 3 terms you will want to consider factoring by grouping.

For this example we have a polynomial with 4 terms.

$$(x^3 + 7x^2) + (2x + 14)$$

Using the associative property, we are going to group this 4-term polynomial into binomials.

$$x^2(x+7) + 2(x+7)$$

Look for the GCF of each binomial and factor it out.

$$(x+7)(x^2+2)$$

Factor out the common binomial.

Example 3

Grouping

$$(a^3 - 2a^2) + (5a - 10)$$

$$a^2(a-2) + 5(a-2)$$

$$(a-2)(a^2+5)$$

Factoring Tool #3

Factor by Substitution

Sometimes Factoring by GCF will not work or make this problem easier.
Substitution allows us to condense what we are working with.

$$x^4 + 8x^2 + 12$$

$(x^2)^2 + 8(x^2) + 12$

let $Z = x^2$

$Z^2 + 8Z + 12$

$(Z+2)(Z+6)$ $2 \times 6 = 12$, 8

$(x^2+2)(x^2+6)$

Since that $x^4 = (x^2)^2$ we are going to choose a variable to substitute for x^2.

Apply your substitution to your problem and factor your new polynomial.

After factoring, REMEMBER TO REVERSE YOUR SUBSTITUTION!

Example 4

$$49x^2 + 35x + 6$$

$49x^2 = 7x \cdot 7x = (7x)^2 = y^2$

$35x = 5 \cdot 7x = 5y$

let $y = 7x$

$y^2 + 5y + 6$

$(y+3)(y+2)$ $3 \times 2 = 6$, 5

$(7x+3)(7x+2)$

Factoring Tool #4

Splitting the Linear Terms (AKA the AC Method)

With this method, we are going to "split" our 2nd term into two terms so that we can factor by grouping.

$$6x^2 + 5x - 6$$

$6 \cdot -6 = -36$

$-4 \quad 9$

5

$(6x^2 - 4x) + (9x - 6)$

$2x(3x - 2) + 3(3x - 2)$

$(3x - 2)(2x + 3)$

Multiply the first and last terms together.

Now we bring back our X-factoring. We want the terms, when multiplied together to give us our product, but when we add them will give us our second term.

Rewrite the middle terms as a sum of our numbers we found.

Now we apply factor by grouping.

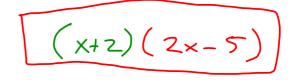For more detailed step-by-step instructions they are provided on page S.23 in Module 4.

Example 5

$$2x^2 - 1x - 10$$

$2 \cdot -10 = -20$

$4 \quad -5$

-1

$(2x^2 + 4x) - (5x + 10)$

$2x(x + 2) - 5(x + 2)$

AC method

$1 \cdot 20$

$4 \cdot 5$

$(x + 2)(2x - 5)$

Example 6

$$6x^2 + 7x - 20$$

$6 \cdot -20 = -120$

$-8 \quad 15$

$(6x^2 - 8x) + (15x - 20)$

$2x(3x - 4) + 5(3x - 4)$

$(3x - 4)(2x + 5)$

AC Method

1. 120   5. 24
2. 60   6. 20
3. 40   7
4. 30   8 · 15

Now factor by Grouping

Example 7

$$-4x^2 + 4x - 1$$

• Can't factor GCF
• Substitution won't help
• Use AC method

$-4 \cdot -1 = 4$

2 2
4

1 · 4 x
2 · 2 ✓

→ • We have 4 terms now, so we can use factor by grouping.

$-4x^2 + 2x + 2x - 1$

$(-4x^2 + 2x) - (2x - 1)$

$2x(-2x + 1) + (-1)(-2x + 1)$

$(-2x + 1)(2x - 1)$

• Factor out a common $(-2x + 1)$

• Write what is left over